Distributed implementation of control barrier functions for multi-agent systems

Xiao Tan and Dimos V. Dimarogonas

KTH Royal Institute of Technology, Sweden

ACC 2022

June 10, 2022

Swedish Research Council

Multi-agent system setting

- Agent dynamics

$$\dot{\boldsymbol{x}}_i = \mathfrak{f}_i(\boldsymbol{x}_i) + \mathfrak{g}_i(\boldsymbol{x}_i)\boldsymbol{u}_i, i \in \mathcal{I} = \{1, 2, ..., N\}$$

  Stacked dynamics

$$\dot{\boldsymbol{x}} = \mathfrak{f}(\boldsymbol{x}) + \mathfrak{g}(\boldsymbol{x})\boldsymbol{u}.$$

- Communication graph $\mathcal{G} = (\mathcal{I}, E)$ is connected and undirected.
  - L: the associated Laplacian matrix;
  - N_i: the neighboring set of agent i;
  - $\boldsymbol{x}_{loc,i}$: the stacked locally available state.

- Safety criterion as a constraint on the stacked state $\boldsymbol{x}$

$$\mathcal{C} = \{\boldsymbol{x} \in \mathbb{R}^n : h(\boldsymbol{x}) \geq 0\}.$$

Multi-agent system setting

- Agent dynamics

$$\dot{\boldsymbol{x}}_i = \mathfrak{f}_i(\boldsymbol{x}_i) + \mathfrak{g}_i(\boldsymbol{x}_i)\boldsymbol{u}_i, i \in \mathcal{I} = \{1, 2, ..., N\}$$

  Stacked dynamics

$$\dot{\boldsymbol{x}} = \mathfrak{f}(\boldsymbol{x}) + \mathfrak{g}(\boldsymbol{x})\boldsymbol{u}.$$

- Communication graph $\mathcal{G} = (\mathcal{I}, E)$ is connected and undirected.
  - L: the associated Laplacian matrix;
  - N_i: the neighboring set of agent i;
  - $\boldsymbol{x}_{loc,i}$: the stacked locally available state.

- Safety criterion as a constraint on the stacked state $\boldsymbol{x}$

$$\mathcal{C} = \{\boldsymbol{x} \in \mathbb{R}^n : h(\boldsymbol{x}) \geq 0\}. \Leftarrow \text{Enforce safety by control barrier functions}$$

Control barrier function for MAS

A CBF-induced controller:

$$\min_{\boldsymbol{u}\in\mathbb{R}^m} \sum_{i\in\mathcal{I}} \frac{1}{2}\|\boldsymbol{u}_i - \boldsymbol{u}_{nom,i}(\boldsymbol{x}_{loc,i})\|^2 \tag{1}$$

$$s.t.\ L_{\mathfrak{f}}h(\boldsymbol{x}) + L_{\mathfrak{g}}h(\boldsymbol{x})\boldsymbol{u} + \alpha(h(\boldsymbol{x})) \geq 0. \tag{2}$$

$\boldsymbol{u}_{nom,i}(\boldsymbol{x}_{loc,i})$: MAS task-related distributed coordination protocol.

Assumption on $h(\boldsymbol{x})$: (2) can be re-written as

$$\sum_{i\in\mathcal{I}} \boldsymbol{a}_i^\top(\boldsymbol{x}_{loc,i})\boldsymbol{u}_i + \sum_{i\in\mathcal{I}} b_i(\boldsymbol{x}_{loc,i}) \leq 0. \tag{3}$$

Control barrier function for MAS

A CBF-induced controller:

$$\min_{\boldsymbol{u}\in\mathbb{R}^m} \sum_{i\in\mathcal{I}} \frac{1}{2}\|\boldsymbol{u}_i - \boldsymbol{u}_{nom,i}(\boldsymbol{x}_{loc,i})\|^2 \tag{1}$$

$$s.t.\ L_{\mathfrak{f}} h(\boldsymbol{x}) + L_{\mathfrak{g}} h(\boldsymbol{x})\boldsymbol{u} + \alpha(h(\boldsymbol{x})) \geq 0. \tag{2}$$

$\boldsymbol{u}_{nom,i}(\boldsymbol{x}_{loc,i})$: MAS task-related distributed coordination protocol.

Assumption on $h(\boldsymbol{x})$: (2) can be re-written as

$$\sum_{i\in\mathcal{I}} \boldsymbol{a}_i^\top(\boldsymbol{x}_{loc,i})\boldsymbol{u}_i + \sum_{i\in\mathcal{I}} b_i(\boldsymbol{x}_{loc,i}) \leq 0. \tag{3}$$

Key question: how to distribute the calculation on $\boldsymbol{u}_i$ while always satisfying (3)?

Distributed implementation

S1. Endows agent i an extra scalar variable y_i and $\boldsymbol{y} := (y_1, y_2, ..., y_N)$.

S2. For a given $\boldsymbol{y}$, define $c_i, i \in \mathcal{I}$ as

$$c_i = \frac{1}{\boldsymbol{a}_i^\top \boldsymbol{a}_i}(\boldsymbol{l}_i \boldsymbol{y} + \boldsymbol{a}_i^\top \boldsymbol{u}_{nom,i} + b_i).$$

and $\boldsymbol{c} := (c_1, c_2, ..., c_N)$.

S3. For agent i, solve local QPs

$$\begin{aligned} \boldsymbol{u}_i &= \arg\min_{\boldsymbol{u}_i \in \mathbb{R}^{m_i}} \frac{1}{2}\|\boldsymbol{u}_i - \boldsymbol{u}_{nom,i}\|^2 \\ s.t.\ & \boldsymbol{a}_i^\top \boldsymbol{u}_i + \sum_{j \in N_i}(y_i - y_j) + b_i \leq 0, \end{aligned} \tag{4}$$

S4. $\boldsymbol{y}$ is updated by $\dot{\boldsymbol{y}} = -k_0 \text{sign}(L\boldsymbol{c})$

Distributed implementation

S1. Endows agent i an extra scalar variable y_i and $\boldsymbol{y} := (y_1, y_2, ..., y_N)$.

S2. For a given $\boldsymbol{y}$, define $c_i, i \in \mathcal{I}$ as

$$c_i = \frac{1}{\boldsymbol{a}_i^\top \boldsymbol{a}_i}(\boldsymbol{l}_i \boldsymbol{y} + \boldsymbol{a}_i^\top \boldsymbol{u}_{nom,i} + b_i).$$

and $\boldsymbol{c} := (c_1, c_2, ..., c_N)$.

S3. For agent i, solve local QPs

$$\begin{aligned} \boldsymbol{u}_i &= \arg\min_{\boldsymbol{u}_i \in \mathbb{R}^{m_i}} \frac{1}{2}\|\boldsymbol{u}_i - \boldsymbol{u}_{nom,i}\|^2 \\ s.t.\ & \boldsymbol{a}_i^\top \boldsymbol{u}_i + \sum_{j \in N_i}(y_i - y_j) + b_i \leq 0, \end{aligned} \tag{4}$$

S4. $\boldsymbol{y}$ is updated by $\dot{\boldsymbol{y}} = -k_0 \text{sign}(L\boldsymbol{c})$

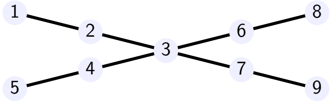
Theorem. Under mild assumptions on k_0 and $\boldsymbol{a}_i$, we have

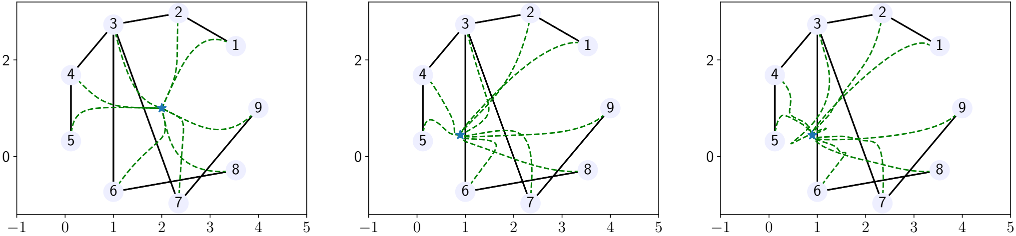
1. the solution to local QPs is identical to that of centralized QP in finite time;
2. the coupling constraint in the centralized QP is satisfied for all time.

Simulations

MAS task: consensus, which leads to $\boldsymbol{u}_{nom,i} = \sum_{j \in N_i} (\boldsymbol{x}_j - \boldsymbol{x}_i)$;

MAS safety constraint: $\{\boldsymbol{x} \in \mathbb{R}^{18} : h(\boldsymbol{x}) = 9 - \boldsymbol{x}^\top \boldsymbol{x} \geq 0\}$





(a) Case 1: nominal controller.

(b) Case 2: centralized CBF controller.

(c) Case 3: distributed CBF controller.

Questions

For more details, find me at

FrB16.7

or, reach me via

`xiaotan@kth.se`