Construction of control barrier function and C^2 reference trajectory for constrained attitude maneuvers

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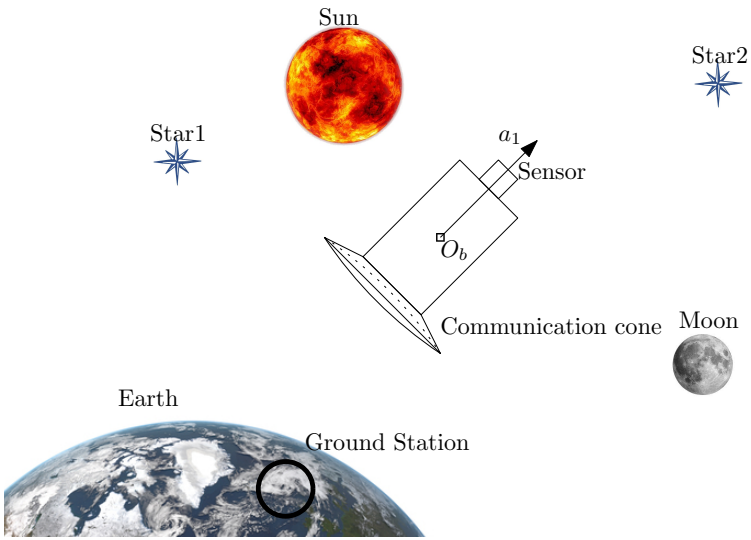
IEEE-CDC 2020

Motivation

Constrained attitude maneuvers



Spacecraft maneuvering while avoiding bright objects and always maintaining communication

Spacecraft docking with ISS

credit: NASA TV

Solution Overview

Dynamics

Configuration manifold

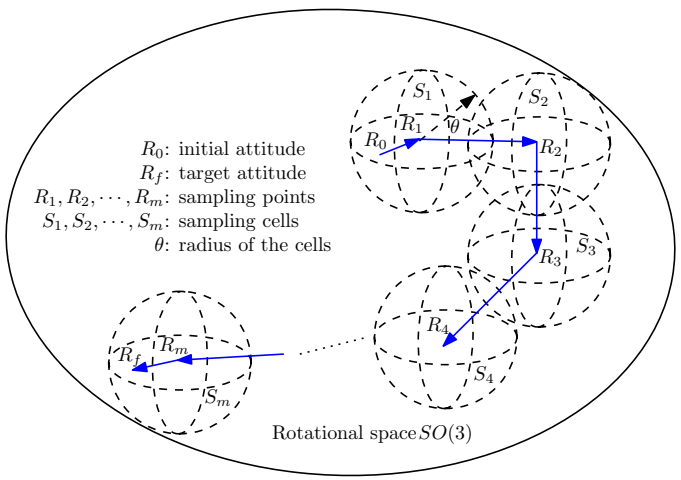
Partitioning + Planning

$\{S_i\}$

$S_1 S_2 S_3 \cdots S_m$

Control scheme

C^2 reference curve construction



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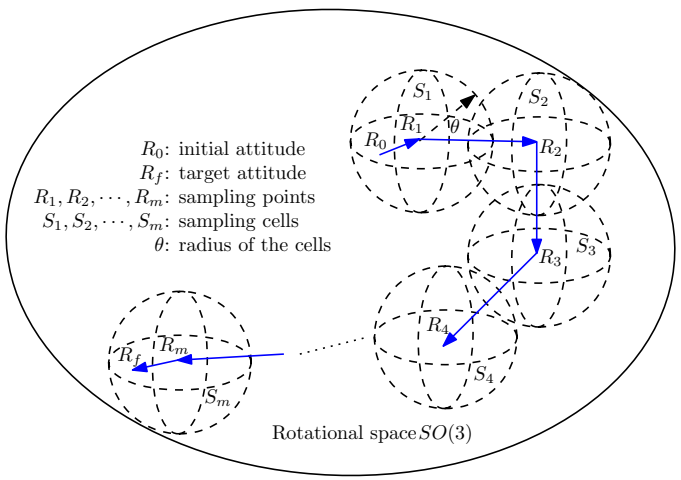
High-order CBF construction



Problem illustration

- attitude dynamics

$$\begin{cases} \dot{R} = R[\omega]_\times, \\ J\dot{\omega} + [\omega]_\times J\omega = u, \end{cases} \quad \text{with } [\omega]_\times := \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}. \qquad (1)$$

[1] **Xiao Tan**, Soulaimane Berkane, and Dimos V. Dimarogonas. Constrained attitude maneuvers on $SO(3)$: Rotation space sampling, planning and low-level control. *Automatica*, 112, 2020.

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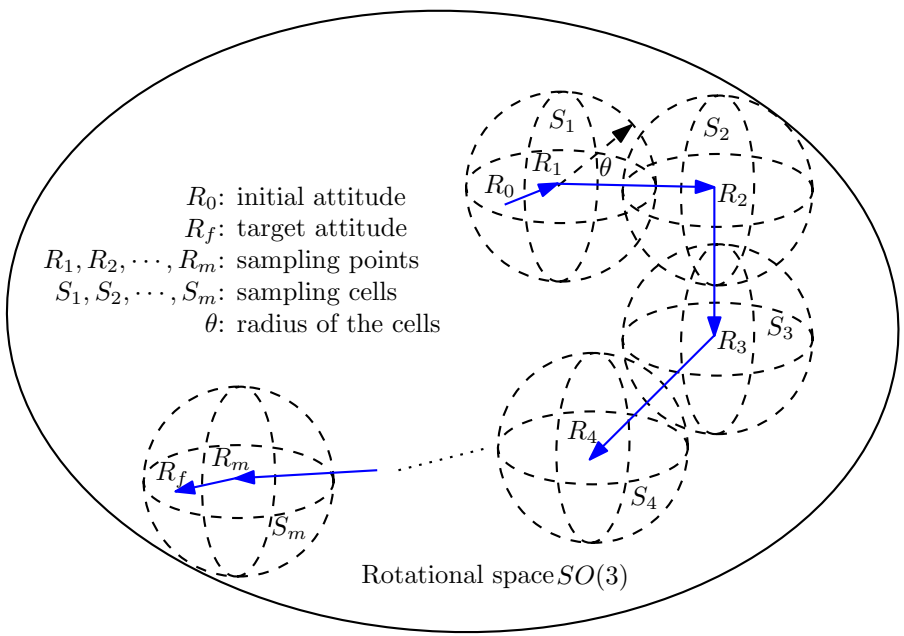
Sampling set
$U := \{R_1, R_2, \ldots, R_i, \ldots, R_n\}$.

Index set $\mathcal{N} = \{1, 2, \cdots, n\}$.

Cell region $S_i := \{R \in SO(3) : \mathrm{d}(R, R_i) < \theta, i \in \mathcal{N}\}, \theta \in (0, \pi/2)$.

The neighborhood set N_i of R_i
$N_i := \{R \in U : \mathrm{d}(R, R_i) < 2\theta, R \neq R_i, i \in \mathcal{N}\}$.



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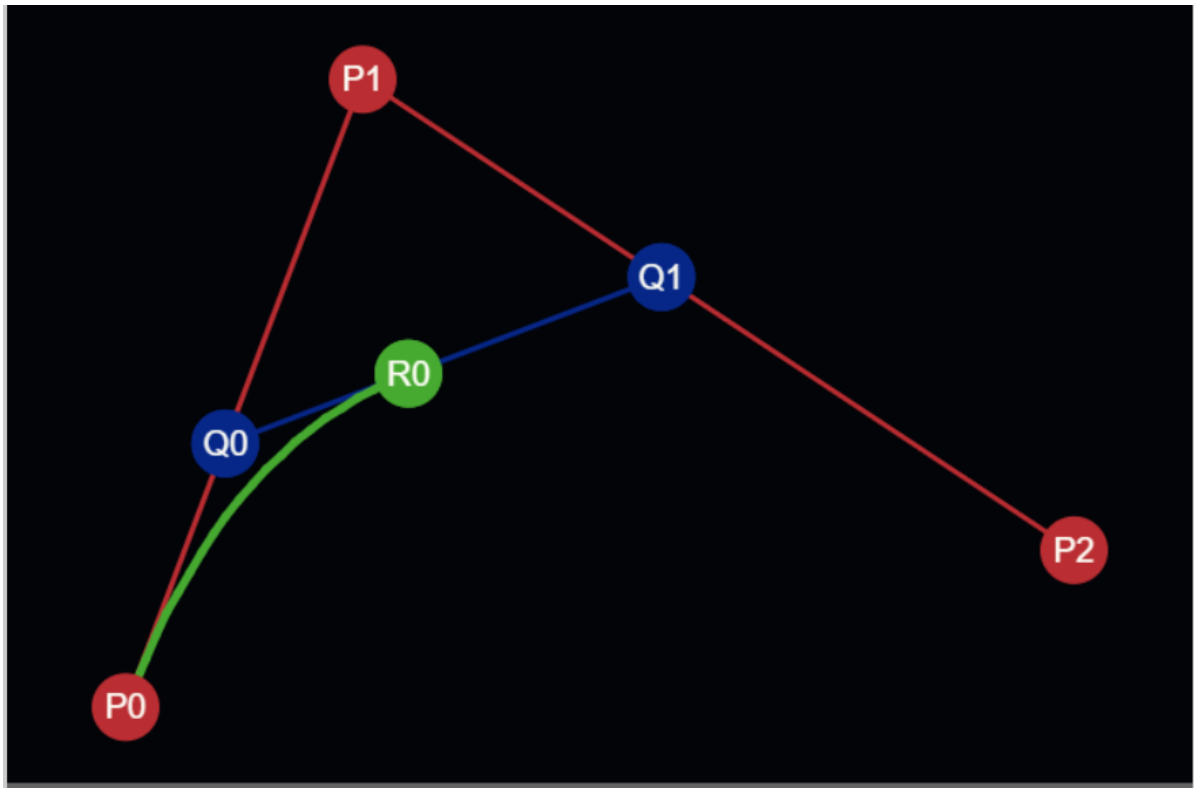
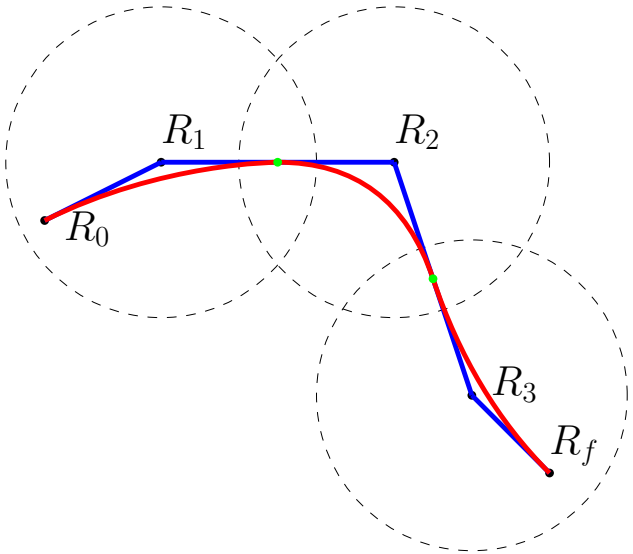
Reference generation: Bézier curve

Bézier curve: **recursive linear interpolation**



http://www.malinc.se/m/DeCasteljauAndBezier.php.

Reference generation: Bézier curve

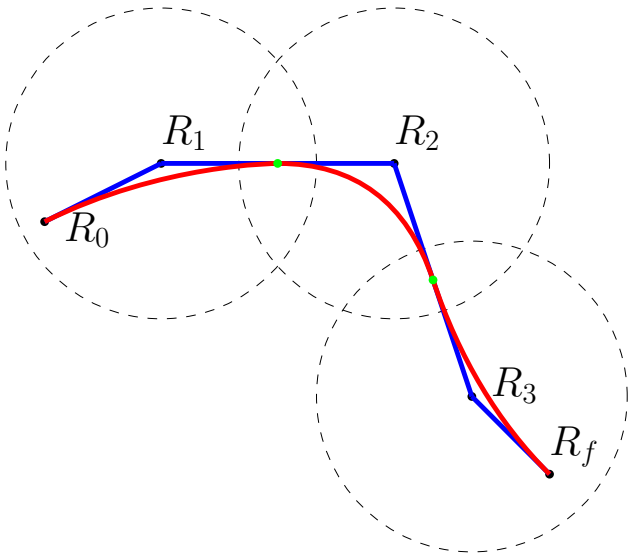


A concatenated Bézier curve $c : [0, m] \to SO(3)$ is defined as

$$c(\tau) = \begin{cases} c_{R_0,R_1,R_{1,2}}(\tau) & \tau \in [0,1), \\ c_{R_{i-1,i},R_i,R_{i,i+1}}(\tau - i + 1) & \tau \in [i-1,i), \\ & i \in \{2,3,\cdots,m-1\}, \\ c_{R_{m-1,m},R_m,R_f}(\tau - m + 1) & \tau \in [m-1,m], \end{cases}$$

where $R_{i,i+1} := R_i \exp(1/2 \log(R_i^\top R_{i+1}))$.

Reference generation: Bézier curve

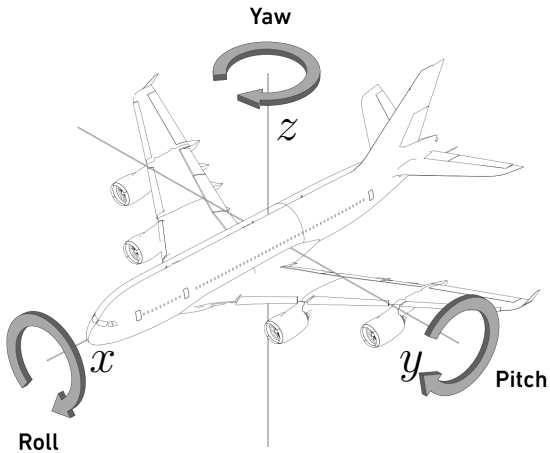


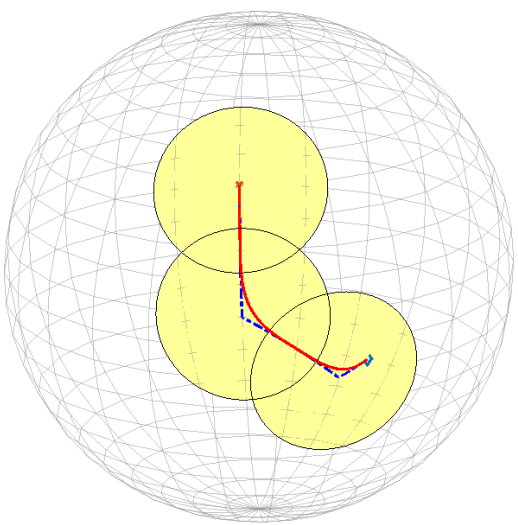
The Bézier curve has the following properties:

- $c(0) = R_0, c(m) = R_f$;
- $c(\tau) \in \cup_{i=1}^{m} S_i$ for $\tau \in [0, m]$;
- $c(\tau)$ is a C^2 curve;

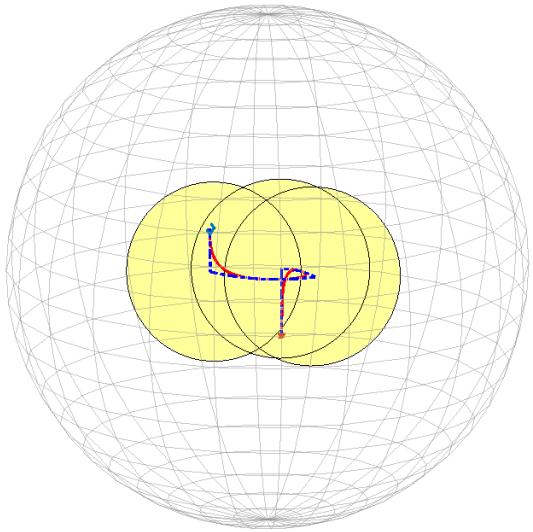
*To obtain the reference trajectory (parametrized with time t), we need re-parametrize the paths with $\tau(t) : [0, T_f] \to [0, m]$, which is neglected here.

Simulation: Geodesic path v.s. Bézier curve

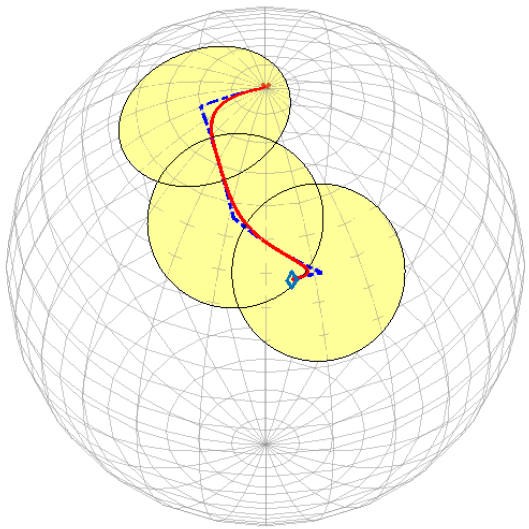




Trajectory of x-axis.



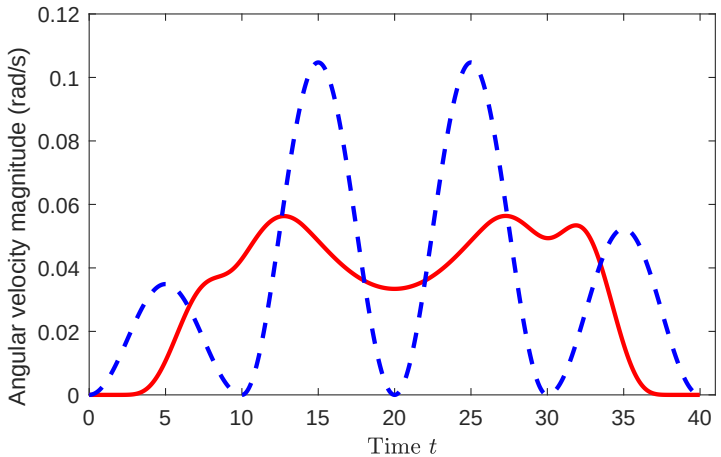
Trajectory of y-axis.

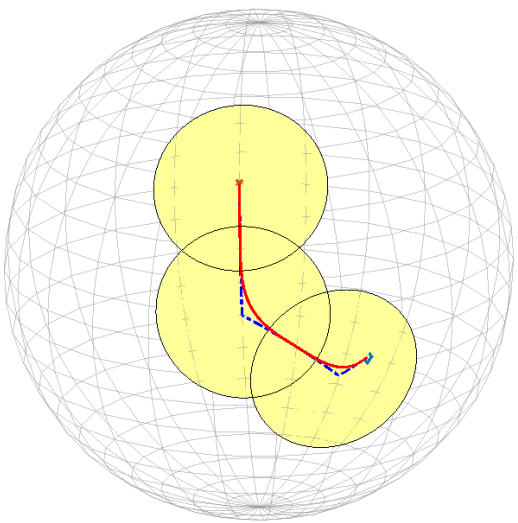


Trajectory of z-axis.

Blue: geodesic path; Red : Bézier curve.

Simulation: Geodesic path v.s. Bézier curve





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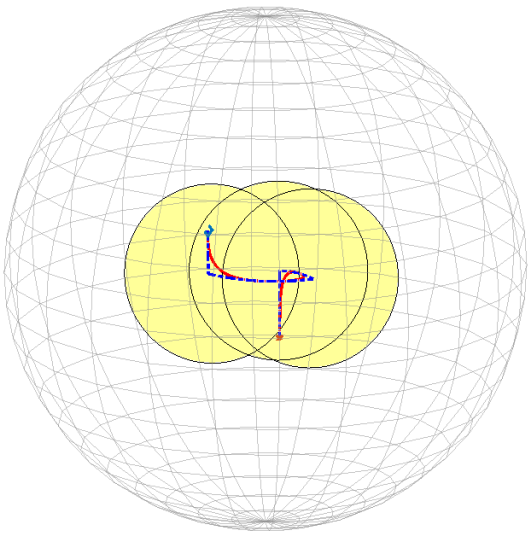

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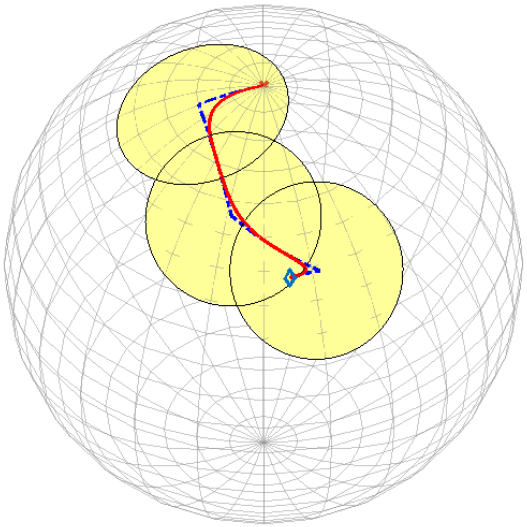

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Overview

Dynamics

Configuration manifold

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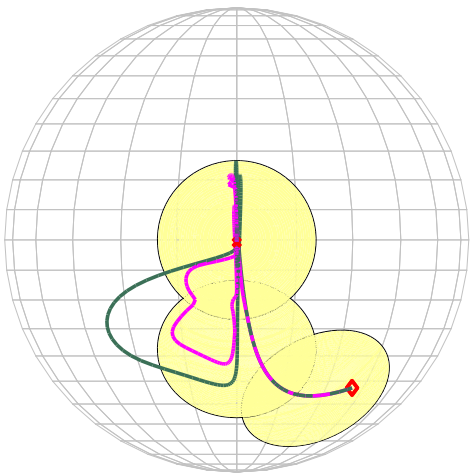
Control scheme

High-order CBF construction

- Non-smoothness
- Possible singularities



Control barrier function design

Define $r_i : SO(3) \to \mathbb{R}$

$$r_i(R) = \epsilon - \|R_i - R\|_F^2/2. \tag{2}$$

with $\epsilon := 4\sin^2(\theta/2)$.

- $r_i(R) > 0$ iff $R \in S_i$.

[2] P. Glotfelter, J. Cortes, and M. Egerstedt, Nonsmooth barrier functions with applications to multi-robot systems, IEEE control systems letters, 2017.

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$$R(t) \in \cup_{i\in\mathcal{N}} S_i \quad \Leftrightarrow \quad \max_{i\in\mathcal{N}}(r_i(R(t))) > 0, \quad \text{for } t \geq 0. \tag{3}$$

max operation $\Rightarrow$ nonsmooth analysis and a complex formulation[2].

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CBF design: Non-smoothness

- Define

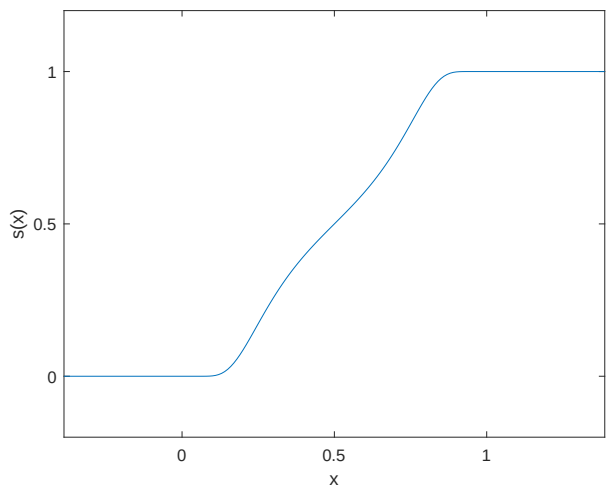
$$h(R) = \sum_{i \in \mathcal{N}} s(r_i(R)/\epsilon) - \delta, \tag{4}$$

where constant $\delta > 0$.

Here

$$s(x) = \begin{cases} 0 & x \in (-\infty, 0] \\ \frac{\rho(x)}{\rho(x)+\rho(1-x)} & x \in (0,1), \\ 1 & x \in [1, \infty) \end{cases}$$

with $\rho(x) := (1/x)e^{-1/x}$.



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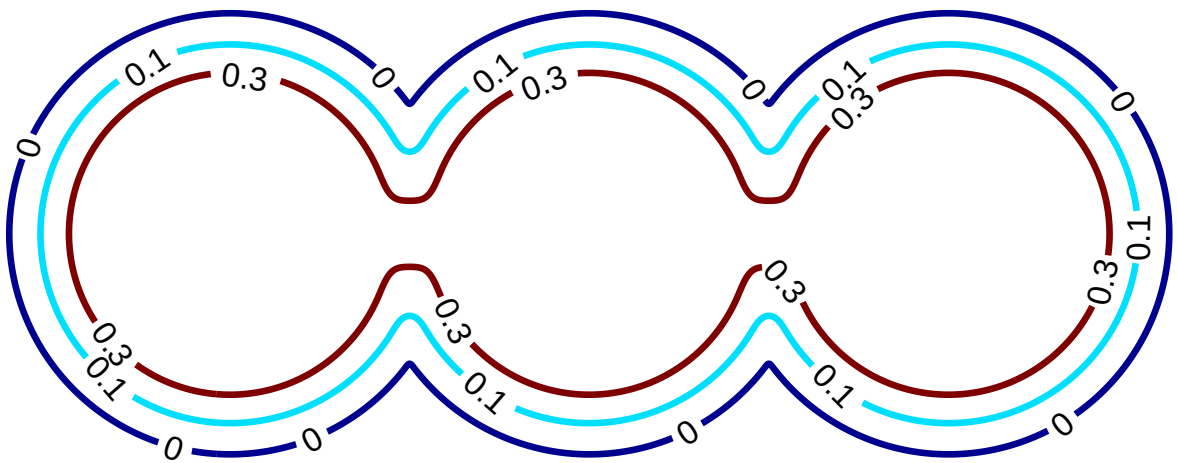

Illustration of C_h in the planar case with different conservativeness δ's.

CBF design: High-order systems

To comply with the control affine form, rewriting the attitude dynamics, we have

$$\dot{x} := f(x) + gu,$$

where $x = \begin{pmatrix} r_{11} \\ r_{12} \\ \cdots \\ r_{33} \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \in \mathbb{R}^{12}, f(x) = \begin{pmatrix} r_{12}\omega_3 - r_{13}\omega_2 \\ r_{13}\omega_1 - r_{11}\omega_3 \\ r_{11}\omega_2 - r_{12}\omega_1 \\ r_{22}\omega_3 - r_{23}\omega_2 \\ r_{23}\omega_1 - r_{21}\omega_3 \\ r_{21}\omega_2 - r_{22}\omega_1 \\ r_{32}\omega_3 - r_{33}\omega_2 \\ r_{33}\omega_1 - r_{31}\omega_3 \\ r_{31}\omega_2 - r_{32}\omega_1 \\ J^{-1}(-[\omega]_\times J\omega) \end{pmatrix}, g = \begin{pmatrix} 0_{9\times 3} \\ J^{-1} \end{pmatrix}.$

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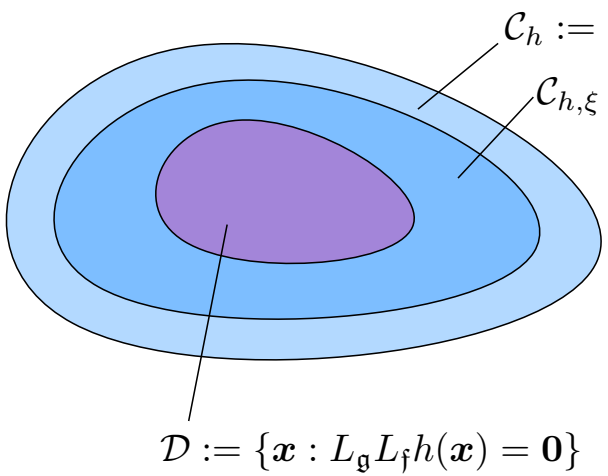
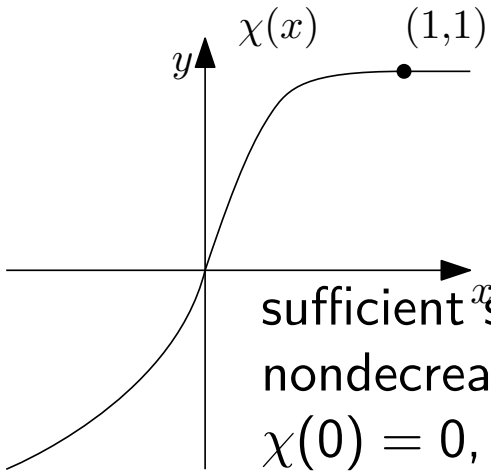
$L_g h(x) = 0$ for all x. Cannot apply ZBF directly.

CBF design: Intuitions behind singularity-free HOCBF

Prop.3: $\mathcal{D} \subset \mathcal{C}_{h,\xi}$.



$$b(x) := \chi\left(\frac{h(x)}{\xi}\right)$$



sufficient smooth,
nondecreasing,
$\chi(0) = 0$,
$\chi(x) = 1$ for $x \geq 1$.

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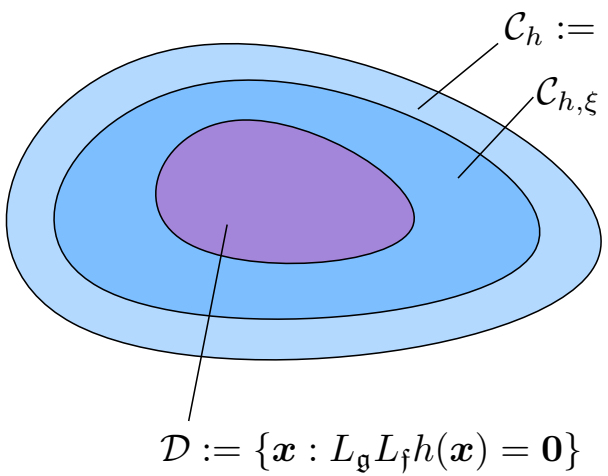

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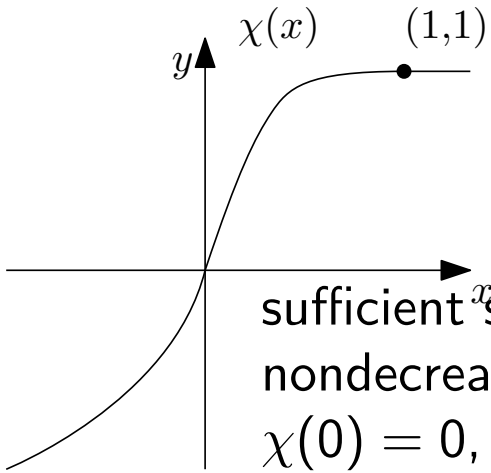

sufficient smooth,
nondecreasing,
$\chi(0) = 0$,
$\chi(x) = 1$ for $x \geq 1$.

- $\mathcal{C}_b = \mathcal{C}_h$.
- For $x \in \mathcal{C}_{h,\xi}$, we derive that, for $1 \leq k \leq r$,

$$b(x) = 1 > 0$$

$$b_1(x) := \dot{b}(x) + \alpha(b(x)) = \alpha(1) > 0$$

$$L_g L_f b(x) u + L_f b_1(x) + \beta(b_1(x)) = \beta \circ \alpha(1) > 0$$

Thus, b is a high-order CBF.

Singularity-free HOCBF

Concretely, the modified controller is given by

$$\begin{aligned} u(x) = \arg\min_{u \in \mathbb{R}^3} \|u - u_{nom}\|^2 \\ \text{s.t.} \quad L_g L_f b(x) u + L_f b_1(x) + \beta(b_1(x)) \geq 0. \end{aligned} \tag{5}$$

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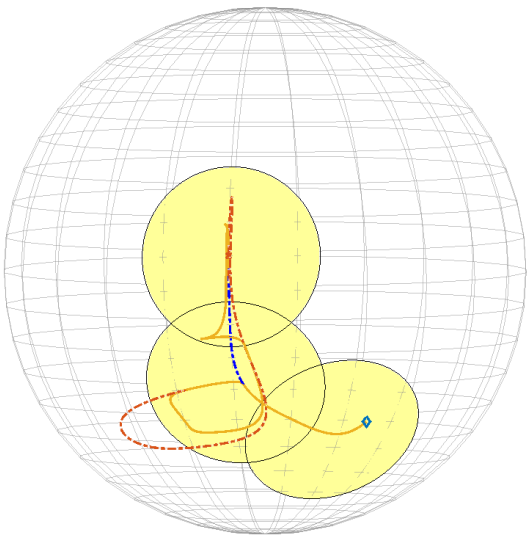
Theorem

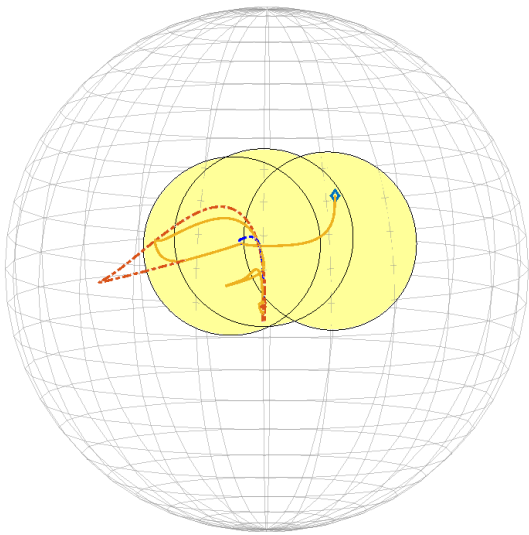
For the attitude control system in (1), the controller (5) renders the set $\mathcal{C}_b \cap \mathcal{C}_{b_1}$ forward invariant.

Simulation: Reference tracking controller

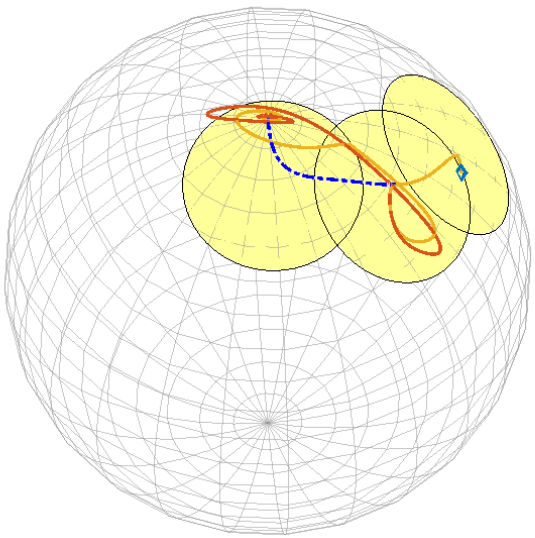
An additive control signal $u_{add}(t)$ for $t \in [20, 25]$ is used in Red and Orange Cases to imitate human inputs to the system.



Trajectory of x-axis.



Trajectory of y-axis.



Trajectory of z-axis.

Blue: Reference tracking control with no additive signal; control barrier function exists;

Red: Reference tracking control with additive signal; control barrier function does not exist;

Orange: Reference tracking control with additive signal; control barrier function exists.

Conclusion

In the talk, we present

- a Bézier curve-based reference trajectory for cell transitions in $SO(3)$, and,

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In the talk, we present

- a Bézier curve-based reference trajectory for cell transitions in $SO(3)$, and,
- a high-order barrier function that ensures the transient performance for cell transitions.

Latest work

High-order control barriers:

- forward invariance and asymptotic stability of the set $\mathcal{C}$;
- singularity-free, performance-critical design.

Xiao Tan, Wenceslao Shaw-Cortez, and Dimos V. Dimarogonas. High-order barrier functions: robustness, safety and performance-critical control. *IEEE Transactions of Automatic Control*, under review.

Thank you for your attention!